Chapter 2

Coulomb Forces and Electric Field Intensity

2.1 COULOMB'S LAW

There is a force between two charges which is directly proportional to the charge magnitudes and inversely proportional to the square of the separation distance. This is *Coulomb's law,* which was developed from work with small charged bodies and a delicate torsion balance. In vector form, it is stated thus,

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\epsilon d^2}\mathbf{a}$$

Rationalized SI units will be used throughout this book. The force is in newtons (N), the distance is in meters (m), and the (derived) unit of charge is the coulomb (C). The system is rationalized by the factor 4π, introduced in Coulomb's law in order that it not appear later in Maxwell's equations. ϵ is the *permittivity* of the medium, with the units $C^2/N \cdot m^2$ or, equivalently, farads per meter (F/m). For free space or vacuum,

$$\epsilon = \epsilon_0 = 8.854 \times 10^{-12}\ \text{F/m} \approx \frac{10^{-9}}{36\pi}\ \text{F/m}$$

For media other than free space, $\epsilon = \epsilon_0 \epsilon_r$, where ϵ_r is the *relative permittivity* or *dielectric constant.* Free space is to be assumed in all problems and examples, as well as the approximate value for ϵ_0, unless there is a statement to the contrary.

For point charges of like sign the Coulomb force is one of repulsion, while for unlike charges the force is attractive. To incorporate this information rewrite Coulomb's law as follows:

$$\mathbf{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2}\mathbf{a}_{21} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^3}\mathbf{R}_{21}$$

where $\mathbf{F}_1$ is the force on charge Q_1 due to a second charge Q_2, $\mathbf{a}_{21}$ is the unit vector *directed from Q_2 to Q_1*, and $\mathbf{R}_{21} = R_{21}\mathbf{a}_{21}$ is the displacement vector from Q_2 to Q_1.

EXAMPLE 1. Find the force on charge Q_1, 20 μC, due to charge Q_2, -300 μC, where Q_1 is at (0, 1, 2) m and Q_2 at (2, 0, 0) m.

Because 1 C is a rather large unit, charges are often given in microcoulombs (μC), nanocoulombs (nC), or picocoulombs (pC). (See Appendix for the SI prefix system.) Referring to Fig. 2-1,

$$\mathbf{R}_{21} = -2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z \qquad R_{21} = \sqrt{(-2)^2 + 1^2 + 2^2} = 3$$

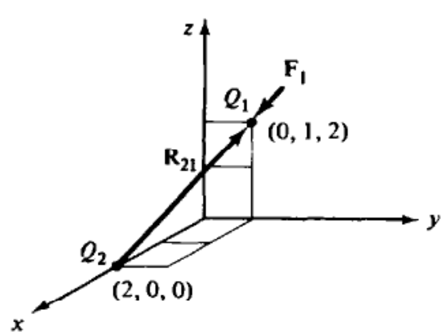

Fig. 2-1

and $$\mathbf{a}_{21} = \frac{1}{3}(-2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)$$

Then $$\mathbf{F}_1 = \frac{(20 \times 10^{-6})(-300 \times 10^{-6})}{4\pi(10^{-9}/36\pi)(3)^2}\left(\frac{-2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z}{3}\right)$$
$$= 6\left(\frac{2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z}{3}\right) \text{N}$$

The force magnitude is 6 N and the direction is such that Q_1 is attracted to Q_2 (unlike charges attract).

This force relationship is bilinear in the charges. Consequently, superposition applies, and the force on a charge Q_1 due to $n - 1$ other charges $Q_2, Q_3, \ldots, Q_n$ is the *vector sum* of the individual forces:

$$\mathbf{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2}\mathbf{a}_{21} + \frac{Q_1 Q_3}{4\pi\epsilon_0 R_{31}^2}\mathbf{a}_{31} + \cdots = \frac{Q_1}{4\pi\epsilon_0}\sum_{k=2}^{n}\frac{Q_k}{R_{k1}^2}\mathbf{a}_{k1}$$

This superposition extends in a natural way to the case where charge is continuously distributed through some spatial region: one simply replaces the above vector sum by a *vector integral* (see Section 2.3).

The force field in the region of an isolated charge Q is spherically symmetric. This is made evident by locating Q at the origin of a spherical coordinate system, so that the position vector $\mathbf{R}$, from Q to a small test charge $Q_t \ll Q$, is simply $r\mathbf{a}_r$. Then

$$\mathbf{F}_t = \frac{Q_t Q}{4\pi\epsilon_0 r^2}\mathbf{a}_r$$

showing that on the spherical surface $r = \text{constant}$, $|\mathbf{F}_t|$ is constant, and $\mathbf{F}_t$ is radial.

2.2 ELECTRIC FIELD INTENSITY

Suppose that the above-considered test charge Q_t is sufficiently small so as not to disturb significantly the field of the fixed point charge Q. Then the *electric field intensity*, $\mathbf{E}$, due to Q is defined to be the force per unit charge on Q_t: $\mathbf{E} = \mathbf{F}_t/Q_t$.

For Q at the origin of a spherical coordinate system [see Fig. 2-2(*a*)], the electric field intensity at an arbitrary point P is, from Section 2.1,

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\mathbf{a}_r$$

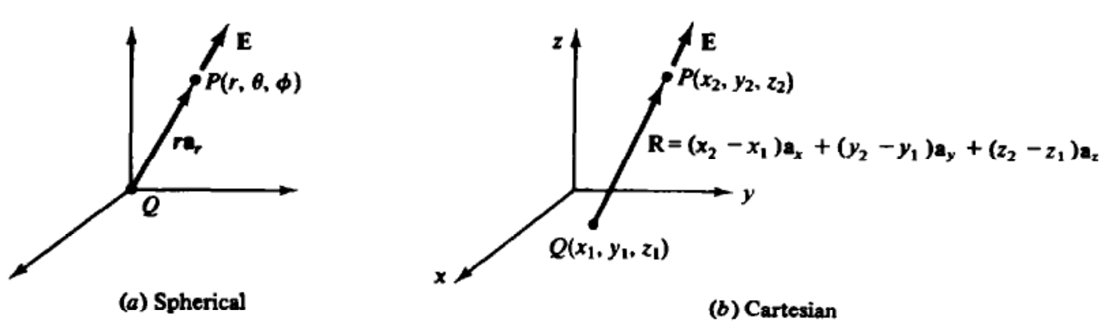

Fig. 2-2